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Einstein gravitation as a long wavelength effective field theory

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A possible resolution of the difficulties in quantizing general relativity is provided by the suggestion that Einstein gravitation is not a fundamental field theory, but rather is a long-wavelength effective field theory, arising as a scale-symmetry-breaking effect in a renormalizable fundamental theory. In unified theories of this type, Newton's constant will be calculable in terms of fundamental particle masses. The history and current status of these ideas is reviewed.

In the conventional picture of the fundamental forces of physics, as recently reviewed in Weinberg (1980), gravitation appears on a quite different footing from the weak, strong and electromagnetic interactions of the matter fields. The total dynamics, in the usual formulation, is governed by an action functional

$$S = \int d^4x(-g)^{\frac{1}{2}}(\mathcal{L}_m + \mathcal{L}_{grav}). \tag{1a}$$

Here $\mathcal{L}_m$ is a renormalizable matter Lagrangian density, containing only dimensionless coupling constants, and $\mathcal{L}_{grav}$ is the Einstein–Hilbert gravitational Lagrangian

$$\mathcal{L}_{grav} = (1/16\pi G)\, R, \tag{1b}$$

with R the scalar curvature. Since the coupling constant $(16\pi G)^{-1}$ appearing in the gravitational action has the dimensionality $(\text{mass})^2$, quantization of the gravitational part of (1) leads to a nonrenormalizable field theory. Furthermore, in the conventional view, there is no mechanism for relating the gravitational mass scale set by $G^{-\frac{1}{2}}$ to the unification mass of the matter fields. Gravitation thus appears as a phenomenon quite outside the usual framework of theoretical ideas on which elementary particle theory is based.

This statement of the problem of 'quantizing gravitation' assumes, however, that the Einstein–Hilbert action is the fundamental quantum action for gravitation. Since all gravitational experiments done to date involve very long wavelengths ($\lambda \gtrsim 10$ cm), there is in fact no experimental evidence for this assumption. Thus, before proceeding to study quantum gravity, we must address the question: is the Einstein theory a fundamental theory, or is it a long wavelength effective field theory?

A familiar example of a long wavelength effective field theory is provided by the Fermi theory of weak interactions, as extended by investigations in particle physics over the last fifteen years. For energies well below 100 GeV (i.e. for wavelengths much longer than 10^{-16} cm), the weak interactions are described by the current–current effective action

$$S_{eff}[(\text{fermions})] = \int d^4x(\mathcal{L}^{ch}_{eff} + \mathcal{L}^{n}_{eff}), \tag{2}$$

where

$$\mathcal{L}^{ch}_{eff} = 2^{-\frac{1}{2}}G_F(j^{\lambda}_{ch} + J^{\lambda}_{ch})(j^{\dagger}_{ch\,\lambda} + J^{\dagger}_{ch\,\lambda}),$$
$$j^{\lambda}_{ch} = \bar{e}\gamma^{\lambda}(1-\gamma_5)\,\nu_e + \mu, \tau \text{ terms},$$
$$J^{\lambda}_{ch} = \bar{u}\gamma^{\lambda}(1-\gamma_5)(\mathrm{d}\cos\theta_C + \mathrm{s}\sin\theta_C) + \mathrm{c}, \mathrm{t}, \mathrm{b} \text{ terms, etc.},$$

with G_F the dimensional constant

$$G_F \approx 10^{-5}/m_p^2, \quad m_p \text{ is the proton mass.} \tag{3}$$

As expected for a theory with a dimensional coupling constant, the Fermi theory is non-renormalizable, and repeated attempts to quantize the weak interactions starting from the Fermi theory as the fundamental quantum action have met with frustration. It is now known that the Fermi theory is only a long-wavelength effective theory for the weak interactions. The fundamental quantum theory for the weak (and electromagnetic) interactions is the renormalizable gauge theory of Glashow, Salam and Weinberg (G.S.W), in which the weak interactions are mediated by the exchange of massive intermediate vector bosons, which obtain their masses from a symmetry-breaking mechanism involving Higgs scalar bosons. When all fermion energies are much lower than the intermediate boson masses, the Fermi theory is recovered from the G.S.W. theory as a low-energy, long-wavelength effective theory for the fermions. In the language of functional integrals, the relation between the Fermi effective theory and the G.S.W. fundamental theory is given by

$$e^{iS_{\text{eff}}[(\text{fermions})]} = \int d(\text{bosons})\, e^{iS_{\text{fund}}[(\text{fermions}),(\text{bosons})]}, \tag{4}$$

and from (4) one readily finds an experimentally verified formula relating the Fermi constant to the parameters of the fundamental theory,

$$2^{-\frac{1}{2}}G_F = e^2/8m_W^2 \sin^2\theta_W, \tag{5}$$

where

$$e = \text{electric charge},$$

$$m_W = \text{charged intermediate boson mass},$$

$$\theta_W = \text{SU}(2)-\text{U}(1) \text{ mixing angle.}$$

Returning now to gravity, let us assume that the strategy that has worked so successfully for the weak interactions should also be applied to the problem of quantizing gravitation. Thus, we shall assume that the fundamental gravitational action is the renormalizable and classically scale-invariant action

$$S_{\text{fund}} = \int d^4x(-g)^{\frac{1}{2}}(\alpha C_{\mu\nu\lambda\sigma}C^{\mu\nu\lambda\sigma} + \beta R^2), \tag{6}$$

with $C_{\mu\nu\lambda\sigma}$ the Weyl tensor (the traceless part of the Riemann curvature tensor $R_{\mu\nu\lambda\sigma}$). Quantum corrections break the scale symmetry of (6), and as a result induce an Einstein–Hilbert effective action in the low-energy, long-wavelength limit; this effective action governs observed gravitational phenomena (just as the Fermi effective theory describes low energy β-decay physics) but is not the fundamental quantum field theory action. The 'induced gravitation' approach just sketched has been actively studied over the last few years, as reviewed in Adler (1982), and is the viewpoint adopted in the remainder of our discussion here.

Continuing to develop the analogy between the Fermi and the Einstein–Hilbert Lagrangians, let us ask what characteristic mass appears in the coupling constant for each effective action. In the microscopic units where $\hbar = c = 1$, the action S is dimensionless, the integration measure d^4x has dimension $(\text{mass})^{-4}$, and thus the Lagrangian density $\mathscr{L}$ has dimension $(\text{mass})^4$. From this fact, and the dimensionalities of the fields appearing in the effective actions, we infer the dimensionalities of the coupling constants already quoted above:

Fermi theory:

$$S_{\text{eff}} = \int \mathrm{d}^4x\, 2^{-\frac{1}{2}} G_{\mathrm{F}} \overbrace{\underbrace{\bar{\psi} \ldots \psi\bar{\psi} \ldots \psi}_{\text{dimension (mass)}^6}}^{\text{dimension (mass)}^4}, \quad \psi = \text{fermion field,}$$

$$\Rightarrow G_{\mathrm{F}} \text{ has dimension (mass)}^{-2}; \tag{7a}$$

Einstein–Hilbert theory:

$$S_{\text{eff}} = \int \mathrm{d}^4x (-g)^{\frac{1}{2}} \overbrace{\frac{1}{16\pi G} \underbrace{R}_{\text{dimension (mass)}^2}}^{\text{dimension (mass)}^4}$$

$$\Rightarrow (16\pi G)^{-1} \text{ has dimension (mass)}^2. \tag{7b}$$

Comparing (7*a*) with (7*b*), we see that there is an important difference between the behaviour of the coupling constant in the two cases. Since the Fermi coupling G_{F} has the dimensionality of mass to a *negative* power, the dominant contributions to G_{F} come from the *smallest* mass intermediate states with the relevant quantum numbers, which are the intermediate bosons with mass $m_{\mathrm{W}} \approx 80\,\mathrm{GeV}$. By contrast, the inverse Newton's constant G^{-1} has the dimensionality of mass to a *positive* power, and hence the dominant contributions to G^{-1} will come from the *largest* characteristic mass scale appearing in physics. This is presumably the Planck mass M_{P} of order $10^{19}\,\mathrm{GeV}$, where the gravitational interactions of elementary particles become of comparable importance to their electro-weak and strong interactions.

Just as G_{F} can be calculated in terms of the more fundamental parameters of the G.W.S. gauge theory, in induced gravity theories one expects Newton's constant G to be calculable in terms of fundamental particle masses. To see how this can come about, we draw on the fact that when $\alpha = g^{-2} > 0$, $\beta = g'^{-2} > 0$, the action of (6) leads to an *asymptotically free* quantum theory, just as the current candidates for unified matter theories are also asymptotically free quantum theories. The term 'asymptotically free' refers to the behaviour of the coupling constant which, when radiative corrections are included, is changed from a true constant to a running function of the dominant dimensional variable, assumed for simplicity in the following discussion to be an energy E. In other words, in an asymptotically free theory, quantum effects lead to the replacement

$$g^2 \rightarrow g^2_{\text{run}} = \frac{g^2(\mu)}{1 + bg^2(\mu) \ln (E/\mu)}, \tag{8}$$

with μ an arbitrary reference mass and with $b(>0)$ a constant characteristic of the theory. As the energy E approaches infinity, (8) implies

$$g^2_{\text{run}} \xrightarrow[E\to\infty]{} 0, \tag{9}$$

which leads to the vanishing of forces, and hence to free field theory behaviour, in the asymptotic high energy limit. On the other hand, the physics described by (8) becomes strongly interacting at low energies, as is readily seen by rewriting (8) in the form

$$g^2_{\text{run}} = \frac{1}{b \ln (E/\mathscr{M})}. \tag{10}$$

Here

$$\mathscr{M} = \mu\, \mathrm{e}^{-1/bg^2(\mu)}, \tag{11}$$

is a mass parameter, which can be shown to be μ-independent, and which characterizes the theory in the sense that E of order $\mathscr{M}$ defines the strong coupling régime. We see that as a result of including radiative corrections, a one parameter family of classical theories characterized by their values of the dimensionless coupling g^2, has been replaced by a one parameter family of quantum theories characterized by their values of the dimension $(\text{mass})^1$ scale mass $\mathscr{M}$. As a result of this phenomenon, called *dimensional transmutation*, all dimensional physical parameters in an asymptotically free theory are calculable in terms of $\mathscr{M}$, much as all radiative effects in the familiar, non-asymptotically free, case of quantum electrodynamics are calculable in terms of the fine structure constant α.

Let us now suppose that the gravitational theory of (6) and the unified matter theories, both of which are asymptotically free, can be further unified into an asymptotically free theory with a single classical coupling constant g. Then as a result of dimensional transmutation, g is replaced as a parameter in the quantized theory by a mass parameter $\mathscr{M}$, which presumably should be identified with M_{P}. All particle masses and G^{-1} will then be calculable in terms of $\mathscr{M}$, or eliminating $\mathscr{M}$, the ratio

$$G^{-1}/(\text{any particle mass})^2, \tag{12}$$

will be calculable. This scenario naturally accommodates the fact that G^{-1} is of order M_{P}^2, but does not explain either why the cosmological constant is very small, or why elementary particle masses are small, on the scale of the Planck mass. These unexplained features are a problem in *all* unified models to date, and presumably will eventually be explained by specific kinematical or dynamical features of the ultimate unifying theory.

Having sketched the principal qualitative features of induced gravity theories, let me now survey in a somewhat more technical way their history and current status.

(1) The suggestion that the Einstein–Hilbert theory is an effective field theory was first made by Sakharov (1967), who proposed that the Einstein–Hilbert action arises from the quantum fluctuations of quantized matter fields in a curved background manifold. (For a survey of this and related early work, see ter Haar *et al.* (1982) and Adler (1982).)

(2) Subsequently, a number of authors (for full references, see Adler 1982) studied Higgs-type models in a curved background manifold, with the action

$$S = \int d^4x(-g)^{\frac{1}{2}}[\tfrac{1}{2}\epsilon\phi^2R - V(\phi) + \dots]. \tag{13a}$$

In (13), $V(\phi)$ is a symmetry-breaking double well potential with a global minimum at $\phi^2 = \bar{\phi}^2$, so that when quantized around the stable vacuum (13*a*) yields an Einstein–Hilbert action, with the gravitational constant given by

$$1/16\pi G = \tfrac{1}{2}\epsilon\bar{\phi}^2. \tag{13b}$$

In models of this type, the renormalized coupling ϵ is necessarily an independent parameter, and so G^{-1} is not calculable in terms of fundamental mass parameters.

(3) Further progress stemmed from the observation (Adler 1980*a*) that in *scalar-free* theories with dynamical breaking of scale invariance, an Einstein–Hilbert action is induced with a calculable Newton's constant G^{-1}. Since a pure non-Abelian gauge theory satisfies these calculability criteria, the simplest model for induced gravity is thus a non-Abelian gauge theory (g.t.) quantized on a classical background manifold, for which the analogue of (4) reads

$$e^{iS_{\mathrm{eff}}[g_{\mu\nu}]} = \int d[A_\lambda]\, e^{iS_{\mathrm{g.t.}}[A_\lambda,\, g_{\mu\nu}]}. \tag{14}$$

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(4) In the background metric model, the coefficients of the various terms in the long-wavelength expansion of S_{eff},

$$S_{\mathrm{eff}}[g_{\mu\nu}] = \int \mathrm{d}^4x(-g)^{\frac{1}{2}}\left[\frac{1}{16\pi G_{\mathrm{i}}}(R-2\Lambda_{\mathrm{i}}) + \alpha C_{\mu\nu\lambda\sigma}C^{\mu\nu\lambda\sigma} + \beta R^2 + \ldots\right], \tag{15}$$

can be extracted by taking successive metric variations. Acting on the left and right sides of (14) with $g_{\mu\nu}\delta/\delta g_{\mu\nu}$ and specializing to flat space–time gives the usual formula for the induced cosmological constant,

$$-(2\pi)^{-1}\Lambda_{\mathrm{i}}/G_{\mathrm{i}} = \langle T^{\mu}_{\mu}\rangle_0, \tag{16}$$

where T^{μ}_{μ} is the trace of the gauge theory stress-energy tensor and where $\langle\,\rangle_0$ denotes the flat space–time vacuum expectation value. Acting with $(g_{\mu\nu}\delta/\delta g_{\mu\nu})^2$ gives a formula (Adler 1980*b*, Zee 1981) for the induced gravitational constant,

$$\left.\begin{aligned} \frac{1}{16\pi G_{\mathrm{i}}} &= \frac{-\mathrm{i}}{96}\int \mathrm{d}^4x\, x^2\langle\mathscr{T}(\tilde{T}(x)\,\tilde{T}(0))\rangle_0, \\ \tilde{T}(x) &= T(x) - \langle T(x)\rangle_0,\ T \equiv T^{\mu}_{\mu}, \end{aligned}\right\} \tag{17}$$

and when higher derivative terms are retained, also a formula (Zee 1982) for the coefficient of the induced R^2 term,

$$\beta = \frac{-\mathrm{i}}{13\,824}\int \mathrm{d}^4x(x^2)^2\langle\mathscr{T}(\tilde{T}(x)\,\tilde{T}(0))\rangle_0. \tag{18}$$

For a general pure non-Abelian gauge theory, the coefficients $\Lambda_{\mathrm{i}}/G_{\mathrm{i}}$, G_{i}^{-1} and β given by (16)–(18) are all calculable in terms of the gauge theory scale mass $\mathscr{M}$.

(5) To go beyond the background metric model, one must take into account the fact that in a realistic theory, gravity (i.e. the metric $g_{\mu\nu}$) is also quantized. To do this, we make the decomposition

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \tag{19}$$

where $\bar{g}_{\mu\nu}$ is the average background metric and where $h_{\mu\nu}$ is a quantum fluctuation. The functional integral formalism for background field quantization then implies that $\bar{g}_{\mu\nu}$ is self-consistently determined by a classical variational principle,

$$\delta S_{\mathrm{eff}}[(\bar{\phi}_{\mathrm{m}}), \bar{g}_{\mu\nu}] = 0, \tag{20}$$

with $\bar{\phi}_{\mathrm{m}}$ the average matter fields. In the long-wavelength limit, (20) yields the classical Einstein equations. The self-consistent structure of the calculation is reflected in the fact that the functional form of S_{m} is itself determined by the quantum fluctuations $h_{\mu\nu}$ around the mean value $\bar{g}_{\mu\nu}$. Acting with $(\bar{g}_{\mu\nu}\delta/\delta\bar{g}_{\mu\nu})^2$ as described above, one obtains a formal functional integral expression for the induced gravitational constant G_{i}^{-1} including quantum gravity effects (Adler 1982).

(6) Finally, let us return to the basic question: what is the fundamental gravitational action? In the absence of additional non-metric degrees of freedom (which may well be present!), the renormalizable candidates are

$$\alpha C_{\mu\nu\lambda\sigma}C^{\mu\nu\lambda\sigma} + \beta R^2, \tag{21a}$$

which is scale invariant, and

$$\alpha C_{\mu\nu\lambda\sigma}C^{\mu\nu\lambda\sigma}, \tag{21b}$$

which is conformally invariant. If the Lagrangian of (21*b*) leads to a finite induced R^2 term (as in the background metric model calculation of (18)) then it is a viable candidate for the fundamental action, while if the induced R^2 term arising from (21*b*) is divergent, then an R^2 counter

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term is needed, as in (21 *a*). The classic objection to the fourth-order Lagrangians of (21) is that, in a small fluctuation analysis, they have an energy spectrum which is unbounded from below. In this connection there is a very interesting new global result (Boulware *et al.* 1983): *for the Lagrangian* (21 *b*), *as well as for* (21 *a*) *with* $\alpha\beta > 0$, *all exact classical solutions have zero energy*. Hence these Lagrangians may well lead in a natural way to satisfactory quantum field theories (without recourse to such unappealing devices as negative metric quantization and special integration contour prescriptions). Thus at this point, the whole subject of the quantization of fourth-order gravitational theories has been re-opened, and is an exciting direction for future research in quantum gravitation.

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